LOYOLA COLLEGE (AUTONOMOUS), CHENNAI – 600 034

M.Sc. DEGREE EXAMINATION – STATISTICS

SECOND SEMESTER – APRIL 2010

ST 2811 / 2808 - ESTIMATION THEORY

Date & Time: 16/04/2010 / 1:00 - 4:00 Dept. No. Max. : 100 Marks

SECTION – A Answer all the questions (10 x 2 = 20)

01. Give an example of a parametric function for which unbiased estimator is unique.
02. Define any two optimality criteria for simultaneous unbiased estimation problem.
03. State a necessary and sufficient condition for an unbiased estimator to be a UMVUE.
04. Explain the importance of Fisher information in finding a sufficient statistic.
05. Give an example of a family of distributions which is not complete.
06. Define ancillary statistic and give two examples..
07. Show that the bias of a location equivariant estimator is free from the parameter.
08. Let X follow B(1, θ), θ = 0.1,0.2. Find the MLE of θ .
09. Define a consistent asymptotically normal estimator and give an example.
10. Deacribe Type I and Type II censoring schemes.

SECTION – B Answer any five questions (5 x 8 = 40)

11. Let X follow DU{1,2,…N}, N = 3,4,… Find the class of unbiased estimators of N .
12. Illustrate the calculus approach to find UMVUE of a parametric function.
13. State Cramer-Rao inequality for the multiparameter case. Find the Cramer-Rao lower bound for estimating σ/μ in the case of $N(\mu,\sigma^2)$, $\mu \varepsilon R$, $\sigma > 0$.
14. Let $X_1,X_2,\dots,X_n$ be a random sample from $U(0, \theta)$, $\theta > 0$. Find a minimal sufficient statistic and examine whether it is complete.
15. State and prove Basu's theorem.
16. Given a random sample of size 2 from DU{ ξ, $\xi + 1$}, $\xi \varepsilon R$, find MREE of ξ with respect to squared error loss.
17. Show that the risk associated with a scale equivariant estimator does not depend on the parameter.
18. Consider a Type II right censored sample from $E(0, \theta)$, $\theta > 0$.Find the maximum likelihood estimator of θ.

SECTION – C Answer any two questions (2 x 20 = 40)

19 a) With usual notations, show that

$$U_g = \{ \delta_0 + u \mid u \varepsilon U_0\},$$

where δ_0 is a fixed member of U_g.

b) State and establish Lehmann – Scheffe theorem.

20 a) Show that an estimator $\boldsymbol{\delta}$ is M – optimal if and only if each component of $\boldsymbol{\delta}$ is a UMVUE.

b) Given a random sample from $E(\mu,\sigma)$, $\mu \varepsilon R$, $\sigma > 0$, find the D – optimal estimator of $(\mu, \mu + \sigma)$ with respect to any loss function, convex in the second argument.

21 a) Discuss the problem of equivariant estimation of percentiles in location-scale models.

b) If the UMVUE of the location parameter exists and location equivariant, show that it is MREE with respect to the squared error loss.

22 a) Discuss an example in which the maximum likelihood estimator is not consistent.

b) Consider a random sample from $P(\theta)$,$\theta > 0$. Assuming a standard exponential prior, find the Bayes estimator of θ with respect to squared error loss.

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